Rationalising surds, Completing the Square

A) Conjugate Surds: $\sqrt{a}+\sqrt{b}$ is the conjugate of $\sqrt{a}-\sqrt{b}$, and vice versa, and $a+\sqrt{b}$ is the conjugate of $a-\sqrt{b}$, and vice versa.

When rationalising surds (i.e. changing the denominator of a fraction to a rational number), multiply the top and bottom of the fraction by the conjugate of the denominator:

Example 1: Write $\frac{\sqrt{6}}{\sqrt{5}+\sqrt{2}}$ with a rational denominator.

Multiply the top and bottom by the conjugate of the denominator, i.e. $\sqrt{5}-\sqrt{2}$.

$$\frac{\sqrt{6}}{\sqrt{5}+\sqrt{2}} = \frac{\sqrt{6}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{6}\sqrt{5}-\sqrt{6}\sqrt{2}}{\sqrt{5}\sqrt{5}-\sqrt{5}\sqrt{2}+\sqrt{5}\sqrt{2}-\sqrt{2}\sqrt{2}} = \frac{\sqrt{30}-\sqrt{12}}{5-2} = \frac{\sqrt{30}-\sqrt{12}}{3}$$

Example 2: Write $\frac{8}{2\sqrt{5}-3}$ with a rational denominator.

Multiply the top and bottom by the conjugate of the denominator, i.e. $2\sqrt{5}+3$.

$$\frac{8}{2\sqrt{5}-3} = \frac{8}{2\sqrt{5}-3} \times \frac{2\sqrt{5}+3}{2\sqrt{5}+3} = \frac{16\sqrt{5}+24}{2\cdot2\cdot\sqrt{5}\sqrt{5}+6\sqrt{5}-6\sqrt{5}-3\cdot3} = \frac{16\sqrt{5}+24}{4\cdot5-9} = \frac{16\sqrt{5}+24}{11}$$

Delta: Exercise 28.6 page 263

B) Completing the Square

Example 1: Complete the square for the expression x^2-6x+5.

$x^2-6x+5 = x^2-6x+\left(\frac{-6}{2}\right)^2+5-\left(\frac{-6}{2}\right)^2$ *adding the square of half the coefficient of x, but also subtracting to balance out the expression*

$= x^2-6x+9+5-9$

$= (x-3)^2 - 4$ *factorising x^2-6x+9 gives you $(x-3)^2$*

Example 2: Complete the square for the expression $3x^2 + 6x - 4$.

$3x^2 + 6x - 4 = 3(x^2 + 2x - \frac{4}{3})$ *taking out the coefficient of x^2 as the common factor*

$= 3(x^2 + 2x + \left(\frac{2}{2}\right)^2 - \frac{4}{3} - \left(\frac{2}{2}\right)^2)$

$= 3(x^2 + 2x + 1 \quad - \frac{4}{3} - 1)$

$= 3((x+1)^2 - \frac{7}{3})$

$= 3(x+1)^2 - 7$

Example 3: Solve $x^2 - 6x + 4 = 0$ by completing the square.

$x^2 - 6x + 4 = 0$

$x^2 - 6x = -4$ *bring constant over to the RHS*

$x^2 - 6x + (\frac{-6}{2})^2 = -4 + (\frac{-6}{2})^2$ *add square of half the coefficient of x to both sides*

$x^2 - 6x + 9 = -4 + 9$

$(x-3)^2 = 5$ *factorising $x^2 - 6x + 9$ gives you $(x-3)^2$*

$x - 3 = \pm\sqrt{5}$

$x = 3 \pm \sqrt{5}$

$x = 5.24 \ \ or \ \ 0.76$

Example 4: Solve $3x^2 - 6x + 1 = 0$ by completing the square.

$3x^2 - 6x + 1 = 0$

$3x^2 - 6x = -1$ *bring constant over to the RHS*

$x^2 - \frac{6}{3}x = \frac{-1}{3}$ *divide both sides by coefficient of x^2*

$x^2 - 2x + (\frac{-2}{2})^2 = \frac{-1}{3} + (\frac{-2}{2})^2$ *add square of half the coefficient of x to both sides*

$x^2 - 2x + 1 = \frac{-1}{3} + 1$

$(x-1)^2 = \frac{2}{3}$

$x - 1 = \pm\sqrt{2/3}$

$x = 1 \pm \sqrt{2/3}$

$x = 1.82 \ \ or \ \ 0.18$

Delta: Exercise 29.2 page 269 Q4, 5.

Homework: Delta Ex 3.3 pg 44 Q26, Ex 3.4 pg 45 Q1-10 (solving log and index eqns)